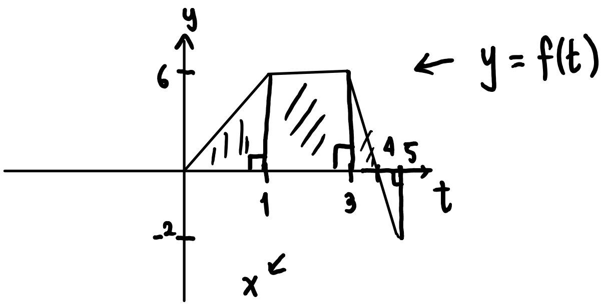
Lecture 24

Friday, November 4, 2016 9:13 AM

The Fundamental Theorem of Calculus

- $g(x) = \int_a^x f(t)\,dt$, f continuous on $[a,b]$
  x varies betn a and b.



$g(x) = \int_0^x f(t)\,dt$. Find $g(0), g(1), g(3)$
$g(4), g(5)$

$$g(0) = \int_0^0 f(t)\,dt = 0$$

$$g(1) = \int_0^1 f(t)\,dt = \frac{1}{2}\cdot 1\cdot 6 = 3$$

$$g(3) = \int_0^3 f(t)\,dt = \int_0^1 f(t)\,dt + \int_1^3 f(t)\,dt$$

$$= 3 + 2.6 = 15$$

$$g(4) = \int_0^4 f(t)\,dt = \int_0^3 f(t)\,dt + \int_3^4 f(t)\,dt$$

$$g(4) = \int_0^4 f(t)\,dt = \int_0^3 f(t)\,dt + \int_3^4 f(t)\,dt$$

$$= 15 + \frac{1}{2} \cdot 1 \cdot 6$$

$$= 18$$

$$g(5) = \int_0^5 f(t)\,dt = \int_0^4 f(t)\,dt + \int_4^5 f(t)\,dt$$

$$= 18 + (-1) = 17$$

FTC Part 1

If f continuous on $[a,b]$, and g defined by $g(x) = \int_a^x f(t)\,dt$, $a \le x \le b$

Then, $g'(x) = f(x)$ for $a < x < b$.

Ex Find derivative of $g(x) = \int_0^x \sqrt{1+t^2}\,dt$.

Since $f(t) = \sqrt{1+t^2}$ is continuous,

$$g'(x) = \sqrt{1+x^2} = f(x)$$

Ex Compute $\dfrac{d}{dx}\left[\int_{x^4}^{1} \cos t\,dt\right]$

$$\left[\int^{x^4} \cos t\,dt\right]$$

$$= \frac{d}{dx}\left[- \qquad \right]$$

Since the upper limit of integration is x^4 we have use Chain rule along with FTC part 1.

Let $u = x^4$,

$$\frac{d}{dx}\left[-\int_1^{x^4} \cos t \, dt\right] = \frac{d}{dx}\left[-\int_1^{u} \cos t \, dt\right]$$

$$= \frac{d}{du}\left[-\int_1^{u} \cos t \, dt\right] \cdot \frac{du}{dx}$$

$$= -\cos u \cdot 4x^3 = -\cos(x^4) \cdot 4x^3$$

- Differentiation & Integration as Inverse Processes

FTC

Suppose f is continuous on $[a, b]$

1) If $g(x) = \int_a^x f(t)\, dt$, then $g'(x) = f(x)$

2) $\int_a^b f(x)\, dx = F(b) - F(a)$, where F ...

2) $\int_a^b f(x)\,dx = F(b) - F(a)$, where F is any antiderivative of f i.e $F' = f$

Rmk Part 2 is called evaluation Thm (Net change Thm).

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

Ex $\displaystyle\int_1^4 \frac{2+x^2}{\sqrt{x}}\,dx$

antiderivative.

First compute $\displaystyle\int \frac{2+x^2}{\sqrt{x}}\,dx$

$$= \int \frac{2}{\sqrt{x}} + \frac{x^2}{\sqrt{x}}\,dx$$

$$= \int 2x^{-1/2} + x^{3/2}\,dx$$

$$= \left[\frac{2x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + \frac{x^{3/2+1}}{3/2+1} + C \right]$$

$$-\tfrac{1}{2}+1 \qquad\qquad 3/2+1$$

$$= \left[\frac{2x^{1/2}}{1/2} + \frac{x^{5/2}}{5/2} + C \right]$$

$$= \left[4x^{1/2} + \frac{2}{5}x^{5/2} + C \right]$$

Then by FTC II,

$$\int_1^4 \frac{2+x^2}{\sqrt{x}} = \left[4x^{1/2} + \frac{2}{5}x^{5/2} \right]_1^4$$

$$= \left[4 \cdot 4^{1/2} + \frac{2}{5}(4)^{5/2} \right] - \left[4 \cdot 1^{1/2} + \frac{2}{5}(1)^{5/2} \right]$$

$$= \left[8 + \frac{2}{5} \cdot 32 \right] - \left[4 + \frac{2}{5} \right] = \frac{82}{5}$$

<u>RmK</u> $\displaystyle \int_a^b f(x)\,dx = \left[\int f(x)\,dx \right]_a^b$

<u>Ex</u> Let $f'(\theta) = \sec\theta \cdot \tan\theta$,

$f(0) = 1$, Find $f(\frac{\pi}{4})$.

$\pi/4$

$f(0) = 1$, find $f(\frac{\pi}{4})$

<u>Soln</u> $\quad f\left(\frac{\pi}{4}\right) - f(0) = \int_0^{\pi/4} \sec\theta \cdot \tan\theta \, d\theta$

$$f\left(\frac{\pi}{4}\right) - 1 = \Big[\sec\theta\Big]_0^{\pi/4}$$

$$f\left(\frac{\pi}{4}\right) - 1 = \sec\left(\frac{\pi}{4}\right) - \sec 0$$

$$f\left(\frac{\pi}{4}\right) = \sqrt{2} - 1 + 1 = \sqrt{2}$$

<u>Average Value of a function</u>

$\{a_1, \dots, a_n\}$, the average $\bar{a} = \frac{1}{n}\sum_{i=1}^{n} a_i$

Let $y = f(x)$, $a \le x \le b$

Then the average value of f on the interval $[a,b]$ is

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$